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A SURVEY OF FATIGUE THEORY

V. S. Ivanova

ABSTRACT

A survey is presented of fatigue theories starting with the work of Ewing and Humphrey published in 1903. This early theory concerning metal wear along a single slip plane was not confirmed experimentally. The thermodynamic theories of Beilby and Haigh involving the transition of metal from the crystalline state to the amorphous state (and vice versa) were also not confirmed experimentally to any noticeable degree. Fatigue theories based on the concept of strain hardening, which followed, did not reflect the specific feature associated with the mechanism of fatigue failure, i.e., did not reflect the difference in failure observed during the action of static and cyclic loading. Fatigue theories of hardening and softening producing metal disintegration under conditions of cyclic loads developed by Soviet investigators and others have served as a further step ahead in the study of the fatigue failure of metals. Subsequent work of Soviet investigators has revealed the importance of taking into account the increase in the resistance to plastic deformation and the decrease in breaking strength in the development of fatigue failure. The statistical theory of fatigue has made it possible to explain the laws which reflect the effect of various factors, particularly of the scale effect, on fatigue criteria. The recent development of dislocation theories has further aided the understanding of fatigue phenomena. These theories show that it is possible to have several mechanisms for the origin of fatigue cracks.

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*Numbers given in margin indicate pagination in original foreign text.

Early Theories

The processes of fatigue failure have been treated rather simply in earlier theories. Initially it was assumed that fatigue failure takes place due to the occurrence of plastic deformation, i.e., in this the limit of fatigue was considered to be the limit of elasticity. This was due to the fact that for certain materials the fatigue limit turned out to be approximately equal to the elastic limit. However, subsequent experimental data showed that the fatigue limit can be greater or less than the elastic limit and that the appearance of traces of plastic deformation does not necessarily indicate the presence of metal fatigue. For this reason it became impossible to indentify the fatigue limit with the elastic limit. In later works deeper physical reasons were considered for explaining the mechanism of fatigue failure.

Ewing and Humphrey (ref. 1) assumed that due to repeated slipping along the same slip line a wearing away of the metal takes place. The products of friction formed in this manner expand the slip bands and cause fatigue cracks to appear. From the standpoint of modern concepts on the mechanism of plastic deformation during the action of a cyclic load, this formulation of the problem is not confirmed. Slip which occurs during the action of repeated load does not take place along the same slip line but rather along neighboring planes; consequently it is impossible to speak of metal wear along a slip plane in this case.

Beilby (ref. 2) developed the "amorphous" theory according to which the metal which lies close to the surfaces of the slip planes undergoes a phase transformation under the action of cyclic loads and becomes an amorphous material with low plasticity and high strength. Fatigue failure occurs due to the origin of tensile stresses when the metal transforms from the crystalline phase into the amorphous phase.

A slightly different formulation of the fatigue failure of metals was proposed by Haigh (ref. 3), who developed the thermodynamic theory of metal fatigue failure. He assumed that the action of cyclic loading causes some particles to transform from a crystalline state into the amorphous state and others to transform from the amorphous state to the crystalline state. At stresses below the fatigue limit this process remains in equilibrium while above the fatigue limit the equilibrium is disrupted and irreversible /4 processes are accumulated which lead to the formation of cavities and cracks.

This theory has not been widely acknowledged since the transition of metal from the crystalline state to the amorphous state (and vice versa) has not been confirmed experimentally to any notable degree.

The Theory of Strain Hardening

In 1923 Gough and Hanson (ref. 4), who based their work on experimental material accumulated by this time and on their own investigations, adopted the following basic premises for constructing fatigue theory: (1) the action of cyclic loads leads to a decrease in the elastic limit and as a result of this the plastic displacements occur at cyclic stresses which are substantially lower than the static elastic limits; (2) for alternating stresses the hysteresis loop is exhibited for stresses both above and below the elastic limit.

Proceeding from these positions, the above authors formulated the fatigue failure process from the standpoint of internal stresses and strain hardening produced by the action of cyclic loads. The plastic displacements occurring in the metal under the action of a cyclic load lead to the cold hardening of the metal and to the redistribution of stresses both inside the grains as well as between them. This makes it possible for plastic deformation to accumulate without an increase in the load. When there is an action of cyclic stresses below the fatigue limit but sufficient for the occurrence of displacement, there is a decrease in the hysteresis loop and a sharp decrease in the magnitude of plastic deformation almost to its complete cessation. However, when the acting stress is above the fatigue limit, the hysteresis loop decreases less effectively and substantial plastic deformation is possible which leads to a limiting case of cold hardening and the occurrence of fatigue cracks.

All these investigations pertain to polycrystalline methods. In subsequent years Gough and other authors undertook the investigation of the fatigue phenomenon on monocrystals and developed the fatigue theory of strain hardening (refs. 5, 6) which, in essence, differed little from the previous theory of fatigue developed for polycrystalline metals. According to this theory the metal near the slip planes is strain hardened under the action of alternating stresses and failure occurs at the instant of time when the strain hardening reaches a critical value, i.e., the process of fatigue failure was formulated in the same manner as under the action of static loads.

According to the theory of strain hardening the fatigue process is divided into three stages: (1) the development of slip which leads to strain hardening, (2) the origin of the fatigue crack, (3) the development of the fatigue crack under the action of stress concentration. The crack is developed until the transverse cross section of the metal reaches dimensions at which the stress becomes equal to the ultimate strength. This has made it possible to explain the presence of two zones at the failure point in the metal: a brittle zone (due to the action of purely fatigue phenomena) and a viscous zone (as in failure under the action of static loads).

The basis for the development of the fatigue theory of strain hardening was the establishment of the fact that there is an increase in hardness as the number of cycles increases. In addition to this it was established that plastic deformation during fatigue is accumulated by slip and twining along the same

crystallographic planes and along the same directions as in the case of static loads. However, it was shown subsequently that unlike static stresses, /5 the slip which occurs during alternating stresses as a result of an alternation in the sign of deformation, does not occur in the same slip plane but rather occurs along adjacent planes (ref. 7).

In order to extend the resulting deductions to polycrystalline metals, investigations were carried out using tricrystalline materials. No new experimental data were obtained pertaining to the role of boundaries on the fatigue process. Therefore, it was assumed that the grain boundaries, during cyclic loads, produce only a retarding action on the development of plastic deformation (analogous to the role of grain boundaries during the action of static loads).

In subsequent works an effort was made to adopt a deeper physical insight to the explanation of the fatigue phenomenon. However, in explaining fatigue theory the authors again based their reasoning from the standpoint of the theory of strain hardening. One of the progessive fatigue theories for its time was the theory of "atomic" strain hardening proposed by Dehlinger (ref. 8), which was based on the heterogeneity associated with the flow of plastic deformation under the action of cyclic loads. Heterogeneous deformation may occur in polycrystalline metal for a series of reasons: (1) the presence of a heterogenous state of stress produced by external forces; (2) a difference in the values of the elastic limit and a difference in the strain hardening of individual grains due to different orientation of grains in the polycrystalline metal; (3) the presence of crystal lattice distortion (in monocrystals); (4) a different degree of stress in the grains on the surface of the metal and inside the metal. All this can lead to internal stresses which, when combined with external stresses, tend to increase the deformation. A later explanation of fatigue failure was associated with the limiting strain hardening due to the action of internal stresses and the distortion of the crystalline lattice. Thus, in its substance, this theory was a further development of the fatigue theory of strain hardening.

Orowan (ref. 9) explained fatigue failure by the local heterogeneity which he arbitrarily represented in the form of small plastic regions imbedded in the elastic matrix. Under these conditions the accumulation of plastic deformation under the action of applied stress may take place only up to a definite value. As the plastic regions become strain hardened, the stress acting in them will increase and if it reaches the ultimate strength level, a crack appears. On the basis of arbitrary assumptions concerning the variation in the strain hardening rate as a function of the applied stress Orowan proposed an equation for the fatigue curve which corresponded to the Veler curve.

The basic advantage of the theory of strain hardening was that it made it possible to explain a series of experimental facts (for example, the observed increase in the cyclic and static strength of metal after the action of cyclic stresses below or slightly higher than the fatigue limit), which could not be explained by earlier theories.

However, the fatigue theory of strain hardening did not reflect the specific features associated with the mechanism of fatigue failure, i.e., it did not reflect the difference in failure observed during the action of static and cyclic loads. On the other hand this difference was clearly established by subsequent X-ray investigations. It was shown (refs. 10, 11) that under the action of cyclic loads the distortion of the crystal lattice is localized to the immediate vicinity of the failure zone whereas during static loads the distortion of the lattice is distributed with approximate uniformity over the entire deformed volume of the metal.

Theories of Hardening and Softening

The special features of the failure process under the action of cyclic loads have been reflected in subsequent theories developed by Soviet scientists. /6

I. A. Oding (ref. 12) was the first to note the complexity of the fatigue process and to point out that the cyclic repetitive stress produces two phenomena which take place simultaneously in a metal: hardening and softening.

Oding examined the association between fatigue processes and the hysteresis phenomenon occurring under the action of deformantion which changes its sign. Analyzing this phenomenon in detail, he examined the special features associated with the loops of elastic and plastic hystereses. The differences in the loops of elastic and plastic hystereses consist of the fact that at the instant when loading and unloading starts, the elastic hysteresis loop is characterized by a delay in the variation of deformation with respect to the variation in the strss (fig. 1, a) while for a plastic hysteresis loop it is characteristic to have a delay in stress with respect to deformation (fig. 1, b). The plastic hysteresis loop also includes the elements of elastic hysteresis. The plastic hysteresis loop is shown in figure 1, b and is observed during cyclic stresses above the fatigue limit.

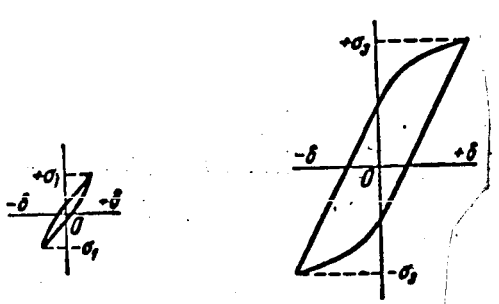

Figure 1. Change in the form of the hysteresis loop as a function of stress (ref. 12).

The width of the hysteresis loop characterizes the cyclic viscocity, i.e., the capacity of the metal to absorb energy in the irreversible form under the action of cyclicly repeated unidirectional stresses or stresses which change their sign.

Whenever these stresses are above the fatigue limit and when the number of cycles increases, the hysteresis loop increases continuously up to the time of failure, as shown in figure 2 (for stresses σ_1 and σ_2). This has made it possible for Oding to assume that at high stress levels, when the width of the hysteresis loop decreases with the increase in the number of cycles, the softening factor predominates over the hardening factor. On the other hand, when the applied cyclic stress is close to the fatigue limit, then according to Oding, two cases can take place:

1. The softening effect is balanced by the hardening effect. Then the width of the hysteresis loop remains constant with the increase in the number of cycles after it has experienced a certain rise as shown in figure 2 when the stress is σ_5.

2. The softening effect first predominates over the hardening effect and then, due to the following intense hardening, the softening factor is suppressed. In this case the width of the loop first increases and then is reduced to a specific limit (fig. 2, for stresses equal to σ_3 and σ_4).

Further, Oding proposed that the cyclic variable plastic deformation produces residual stresses between variously oriented grains, and that at the instant of loading and subsequent unloading there is a redistribution in the residual stresses between grains. This produces a decrease in the proportional limit which results in the widening of the hysteresis loop. Thus, in the case of the cyclic load, Oding considers that stresses of types II and III serve as the softening factors. On the other hand, hardening in the metal is associated with two factors--cold hardening and aging. /7

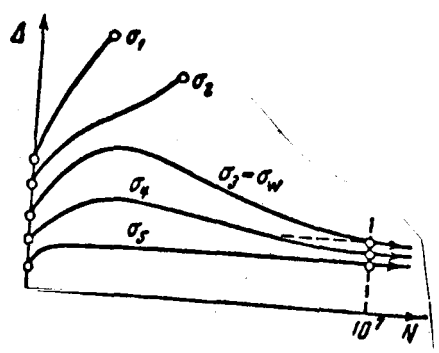

Figure 2. The variation in the cyclic viscocity during the process of cycle increase as a function of stress ($\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4 > \sigma_5$).

The effectiveness of hardening and softening action in the process of cycle accumulation changes and the magnitude of the plastic hysteresis is stabilized (it may even decrease) or increases, depending on the overall action. In the latter case, failure is imminent.

On the basis of this theory it was possible for Oding to:

(1) explain the phenomena of metal conditioning at stresses close to the fatigue limit;

(2) show the association between fatigue limits in tension-compression, flexure and tortion;

(3) establish the relationship between the fatigue limit during uniaxial loading and complex stresses and also under conditions of an assymmetric cycle;

(4) derive the relationship between the fatigue limit of smooth samples and of samples with cuts and to explain the nature of the effect produced by stress concentration.

The fatigue of metal, taking into account the phenomenon of plastic hysteresis, has also been analyzed in the work of Kawamoto and Nischioka (ref. 13). Having examined the special features of the hysteresis loop during the action of cyclid loads, these authors proposed an analytical expression for determining the fatigue limits as a function of load type and sample shape. A comparison of computed and experimental values for the fatugue limits showed good agreement.

All this shows that in the analysis of fatigue processes it is rather useful to take into account plastic hysteresis both from a theoretical as well as from a practical point of view.

In regard to the formulation of the fatigue process, we should point out that the concept of hardening and softening was developed in subsequent investigations in which the softening stage of steel has been called the stage of disintegration. The reasons which cause the disintegration of metal are formulated in a different manner by different authors.

S. Tsobkallo and B. A. Kuznetsov (refs. 14, 15) who investigated the fatigue process by an optical method using silver chloride showed that the development of local disintegration does not cease even when the stresses in the disintegration region become less than nominal. This has served as a serious refutation of the theories of strain hardening. From the point of view of these authors, the reason for fatigue failure is the decrease in the strength of the metal due to disintegration which is preceded by strain hardening. The fatigue stage is associated with the mechanisms of plastic deformation of two types: slip, which leads to strain hardening and the breakup of crystals into blocks which produces the disintegration of the metal.

Glikman and Tekht (ref. 16) have considered the strain hardening /8
stage under the action of cyclic loading as a preparatory phase for the fatigue process which then transforms into the second stage of disintegration at which time fatigue cracks originate and are developed. The fatigue theory based on the concept of strain hardening and disintegration has been developed in the works of N. N. Davidenkov and G. T. Nazarenko (ref. 17), T. A. Lebedev (refs. 18, 19) and others.

The stages of hardening and softening are considered by T. A. Lebedev (ref. 18) as successive stages of fatigue failure. The key role during the stage of metal softening in this case is attributed to the mechanical oscillations of individual microvolumes of the metal. The inteference of mechanical oscillations produced by external forces with low frequency fluctuations of thermal oscillations may lead to the excitation of stresses, which are compatible to the interatomic bonds, and to the occurrence of fatigue cracks in this manner. Various defects in the metal and also geometric stress concentrators serve as sources of local oscillations where the probability of occurrence of oscillations with relatively high amplitudes is large. Pointing to the role of mechanical oscillations in the development of fatigue processes, Lebedev has noted that the Veler curve, from theoretical considerations, may not move parallel to the axis of the abscissas as it is now assumed for ferrous metals, but must have a slope. He also noted that the classification of metals into those which have and which do not have a fatigue limit is rather arbitrary. From Lebedev's point of view, even when the cyclic stresses are small, the interference of oscillations will take place, and consequently it is possible to have stress peaks which lead to the origin of cracks.

Although the mechanism of metal disintegration under conditions of cyclic loads is formulated by the different authors of fatigue theory and disintegration theory from various points of view, this group of theories is a further step ahead in the study of the nature of the fatigue failure of metals.

Breaking Strength and Fatigue

In series of works, fatigue failure is formulated from the standpoint of the role of breaking strength in failure. At the same time the failure mechanism itself is not considered.

G. V. Uzhik (ref. 20), who investigated the conditions of stability from the position of the strength of materials subjected to normal as well as shear stresses, formulated the conditions when strength is disrupted under the action of alternating loads. The key role in his method is attributed to stress concentrators and the action of normal stresses. In the opinion of Uzhik metal fatigue begins in places where stress concentrations exist (cavities, slag inclusions, scratches, etc.). Fatigue failure occurs at the instant of time when the maximum normal stress (σ_1) at the apex of the cut reaches a value corre-

sponding to the breaking strength R_σ (i.e., $\sigma_1 = R_\sigma$). In this connection it is assumed that alternating stresses which exceed the fatigue limit increase the shear strength and decrease the breaking strength.

S. I. Ratner (ref. 21) also holds the point of view that the basic factor which predetermines fatigue failure involves the lowering of the breaking strength during the fatigue process. By using 35 KhGSA steel, Ratner showed experimentally that the decrease in the breaking strength begins long before the appearance of a visible crack and is most pronounced for a high external state of stress. The rate of decrease in the brittle strength for various metals is different and depends on the magnitude of the cyclic viscocity. The higher the cyclic viscocity, with other conditions equal, the sharper is the decrease in the breaking strength and the shorter is the life.

Ratner points out that in the general case we should take into account two processes--the increase in the resistance to plastic deformation and a decrease in breaking strength. The last factor is of prime significance in the development of fatigue failure.

Statistical Theories

For the cyclic strength of metals it is characteristic to have a spread in the experimental data when a large number of samples are tested, for example, at constant stress amplitude. The spread in experimental data is also observed when static strength is investigated, particularly in the case of brittle materials. This has made it necessary to evaluate the strength by statistical means, i.e., by the nethod of averaging out the properties of individual grains or by determining the probability of finding grains with assigned strength characteristics.

The quantitative solutions of the problem associated with the evaluation of brittle strength on the basis of the statistical theory has been carried out by Weibull, Ya. I. Frenkel' and T. A. Kontorova. Thus Weibull, (ref. 22) in solving this problem, based his approach on the dimensions of heterogeneity, assuming that a heterogeneity of the greatest size is particularly dangerous for failure; the relationship between the dimensions of the heterogeneity and the dependent brittle strength has been derived on the basis of the Griffith theory. Ya. I. Frenkel' and T. A. Kontorova (ref. 23), in selecting the parameter which characterizes the heterogeneity, did not take the dimension of the heterogeneity but rather the possible values of sample brittle strength corresponding to these heterogeneities. This has made it possible to give up the necessity of using the Griffith theory in this case.

The premises which have been developed in the statistical theory of strength may be used for formulating a statistical theory of fatigue, particularly since

(as shown in the works of Gatto, reference 24, et al.) in the case of fatigue the amplitude of deviations in the experimental data (in tems of stresses) from the mean value is constant over the entire fatigue curve and the deviation law is sufficiently close to the Gauss law.

N. N. Afanas'yev (ref. 25, 26) developed a stastical theory of fatigue by using certain premises developed in the earlier statistical theories of strength, and the physical concepts concerning the stucture of metals. The author proceeded from the following propositions: (1) a real metal consists of individual crystals which have internal stresses and which, due to their growth, do not constitute a homogeneous mass; (2) the mechanical properties of individual grains are different in the direction of the acting force (due to the presence of chemical heterogeneity and a heterogeneity in the state of stress).

In order to obtain quantitative data, Afanas'yev made a series of simplifying assumptions. The problem was solved by two methods. It was assumed that either all of the grains have the yield point in the direction of the acting force but have different stresses, or that all of the grains are stressed the same way but have a different yield point. The author showed that both of these simplifications are equivalent. Proceeding from these assumptions, he derived an equation for the elongation curve which was in good agreement with experimental data and used these data to compute the probability of the occurrence of fatigue cracks. From the standpoint of Afanas'yev's theory, a fatigue crack occurs in the case when the amplitude of the reduced stress (product of the relative elongation and the modulus of elasticity) for a given grain reaches the breaking strength. Thus the computation of the fatigue limit according to /10
Afanas'yev is reduced to the solution of the problem concerning the probability of finding one or several grains in the metal which are loaded to a stress equal to or higher than the critical value. The author proposed an equation which makes it possible to determine the number of half cycles necessary for obtaining the first crack in the sample. He noted three stages in fatigue failure:

(1) the strain hardening of the crystallites which are subjected to the highest stress;

(2) the disintegration of some of the strain hardened crystallites and the formation of cracks;

(3) the propagation of cracks which have been formed.

The Afanas'yev theory played a positive role and made it possible to describe a series of fatigue laws in a satisfactory manner. However, the idea of the theory concerning the continuous rise of stresses in "weak" grains was not confirmed experimentally (ref. 14).

Freudental (ref. 27) considers that the substantial spread in the fatigue criteria observed experimentally is explained by the statistical distribution of grains with various dimensions in the metal and by the initial position of

the slip bands for a given stress. Having examined the statistical nature of the process associated with the generation and development of fatigue damage inside a heterogeneous polycrystalline structure and basing his reasoning on the progressive destruction of submicroscopic cohesive bonds (ref. 28), Freudental constructed the following statistical theory of fatigue.

At a certain stress amplitude S, the probability that no failure will occur after N cycles is assumed to be equal to

$$l(S)_N = [1 - \pi(S)]^{mN} = [1 - p(S)]^N,$$

where

$$\pi(S) = \int_0^\infty \psi(S) \left[\int_0^S \varphi(r)\, dr \right] dS;$$

m is the number of bonds which must be destroyed to produce a breakage of the samples;

p(S) is the probability function for the strength of the sample;

$\varphi(r)$ is the distribution function for the strength r of cohesive bonds in the sample;

$\varphi(S)$ is the distribution function for the forces of individual bonds when the stress S is applied to the sample.

The equality $[1 - \pi(S)]^m = [1 - p(S)]$ determines the effect of sample size (number of bonds) on t(S).

The most important and yet indeterminate is the probability function p(S) which predetermines the strength of the sample for assigned stresses produced by external forces. To avoid this indeterminacy Freudental, basing his reasoning on the Mises analysis (ref. 29), assumed that when N increases without bounds there are only three limiting functions with extreme values and only one of them is applicable to the fatigue problem (ref. 30). It can be expressed as

$$l(S)_N = \exp\left[-\left(\frac{S - S_{ON}}{S_V - S_{ON}} \right)^{\beta_N} \right], \tag{1}$$

where S_{ON} is the bounded fatigue limit for a given number of cycles N;

β_n is the exponential index which depends on N;

S_V is the stress for which $1(S)_V = 1/e$.

The limiting function when $N = \infty$ for which $S_{ON} = S_O$, expresses the probability of the infinitely long operating life of the metal at the stress S_O--the true fatigue limit.

Since the fatigue tests are usually conducted with a constant amplitude and with a definite number of cycles before failure, the function $1(S)_N$ must be determined from the function $1(N)_S$, which is a rather difficult and complex operation. Theoretical considerations and experimental data make it possible for us to conclude that the nature of extreme distribution for the function $1(N)_S$ is the same as for $1(S)_N$,

$$l(N)_S = \exp\left[-\left(\frac{N - N_{OS}}{V_S - N_{OS}}\right)^{\alpha_S}\right], \tag{2}$$

where V_S is the number of cycles for which $1(V_S) = 1/e$;

α_S is the exponential index which depends on S;

N_{OS} is a constant which determines that minimum number of cycles which produces a typical fatigue failure (as opposed to the statistical case).

By combining equations (1) and (2) we obtain the condition

$$l(N)_S = l(S)_N = \text{const},$$

then

$$\left(\frac{N - N_{OS}}{V_S - N_{OS}}\right)^{\alpha_S} = \left(\frac{S - S_{ON}}{S_V - S_{ON}}\right)^{\beta_N} = \text{const};$$

from which it follows that

$$(V_S - N_{OS})^{\alpha_S} = \text{const}\,(S - S_{ON})^{-\beta_N}$$

or

$$(N - N_{OS})^{\alpha_S} = \text{const}\,(S_V - S_{ON})^{-\beta_N}.$$

These are the basic equations of the Freudenthal theory.

It should be pointed out that in practice it is very difficult to utilize this equation because of the large number of constants which it contains. Also it is not clear how the constants α_S and β_N should be determined. In the first case they depend on the stress; in the second case they depend on the number of cycles.

The statistical theory of fatigue makes it possible to explain the laws which reflect the effect of various factors and first of all the effect of the scale factor on the fatigue criteria. A further development of the statistical fatigue theory is extremely important for the solution of fatigue problems.

Dislocation Theories

In recent years there has been a large development of theories which try to explain the phenomenon of fatigue on the basis of dislocation phenomena in the mechanism of plastic deformation.

The following factors are taken into consideration in the development of these theories:

(1) dislocations occur in metals during crystallization and form a mosaic structure in the annealed monocrystal (units with small disorientation);

(2) dislocations originate during the process of plastic deformation by means of the Frank-Reed mechanism;

(3) when the motion of the dislocation causes it to come up to the surface, it forms a slip line on the surface due to the occurrence of a step of definite magnitude;

(4) as the dislocation moves and encounters an obstacle, vacant places as well as embedded atoms occur in the crystalline lattice;

(5) during the motion of dislocations with an opposite sign in parallel slip planes, the dislocations may annihilate and form cavities.

First let us consider the dislocation theory of fatigue developed by Fujita (ref. 31) for plastic metals. In order to investigate the mechanism of fatigue crack formation, the author considered the interaction of two dislocations of opposite signs situated in parallel planes (fig. 3).

Two such dislocations cannot pass each other, since the interaction force F_x between them in the slip direction is a repulsing force and is given (ref. 31) by the equation

where G is the shear modulus

b is the Burgess vector;

ν is the Poisson ratio;

x is the distance between dislocations in the slip direction;

h is the distance between the slip planes.

The maximum value $F_x = 0.25\ D_{b/h}$ when $x = 0.41$ h. Calculations show that the value of the external stress necessary for overcoming the force F_x must be very large; in the case of copper, for example, the resultant stress must be equal to 60 kg/mm^2 when h = 10Å.

In plastic metals, when we have two rows of retarded dislocations of opposite signs situated in parallel slip planes, the passage of one dislocation over another one is made easier. This is explained by the fact that the newly formed dislocation is subjected to the stress $n\tau$ (ref. 32) where n is the number of dislocations in each row while τ is the tangential stress in the slip direction. In addition to this the overall action of other dislocations situated close to the forward dislocation may decrease the forces of repulsion which act on each forward dislocation.

Further, Fujita assumed that:

(1) the distribution of dislocations into two rows is symmetrical;

(2) the dislocations are in a state of static equilibrium

(3) the forward dislocation A and the one following it, C, (fig. 4) act on two other dislocations D (and vice versa);

(4) the interaction between other dislocations with opposite signs is neglected.

Proceeding from this, Fujita obtained the maximum value for the forces of repulsion acting on each of the forward dislocations equal to $F_A = F_B = 0.109\ Db/h$. When h = 10^{-7} cm the forward dislocations may pass each other when the applied stress is equal to 10^{-4} G, there are 60 dislocations in a row.

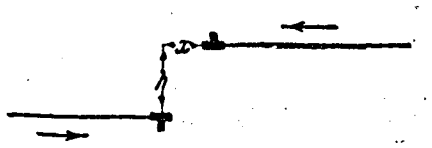

Figure 3. Two parallel unlike linear dislocations situated in different slip planes (ref. 31).

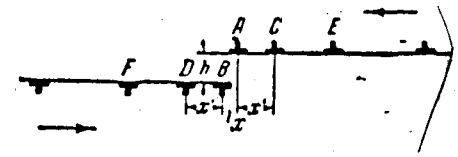

Figure 4. Two parallel unlike series of linear dislocations retarded by each other (ref. 31).

Consequently two forward dislocations of unlike sign may pass by each other even if their slip planes are situated close to each other.

The atomic planes situated perpendicular to the slip planes will be subjected to tensile stresses

$$S_{xx} = 4D/h \approx E/10,$$

where E is the modulus of elasticity. /13

Analyzing the interaction of forward dislocations from this poir of view, Fujita came to the conclusion that if their slip planes are situated a distance $h < 10^{-7}$ cm, then the dislocations will be annihilated and cavi es will be formed. If on the other hand, $h < 10^{-7}$ cm the dislocations may p each other and not unite since the energy of the "hole" in this case will e greater than the energy of the dislocation. The cavity formed as a result o the union of two dislocations will absorb new dislocations and will grow into crack whose width is equal to mb where b is the Burgess vector and m is th number of absorbed dislocations. Further developments of the crack will ta place under the action of stress concentration and when several cracks are ombined in the slip plane a fatigue crack will be formed which leads to fail e.

$$F_x = -D \cdot q \cdot x (x^2 - h^2)/(x^2 + h^2)^2; \quad D = Gb/2\pi(1-\nu),$$

Fujita proposed an equation for the fatigue curve of the form

where k is a constant;

S is the acting stress;

N is the number of cycles before the occurrence of the first crack.

In conclusion, the author has pointed out that in order to construct a more rigid theory of fatigue it is necessary to take into account other factors

which can effect the fatigue process, specifically: the dissociation of dislocations, atomic diffusion, the curvature of the slip plane; interaction between dislocations and vacancies and others.

The role of vacancies in fatigue failure of metals was considered in the works of Oding, who developed the vacancy-dislocation theory of fatigue (ref. 33-35). Oding considered the conditions under which dislocations are displaced during the period of a single symmetric cycle. The motion of dislocations under the action of the applied stress during the first quarter of the cycle leads to the formation of obstacles in the form of vacancies, dislocated atoms, new dislocations and field forces around them. These obstacles retard the return of a part of the dislocations to their sources during the unloading time in the second quarter of the cycle. During the third quarter of the cycle, new loading with an opposite sign takes place. This period is characterized by a smaller resistance to deformation, which is associated with a partial return of dislocations which are displaced and not returned into the initial state during the first quarter of the cycle. The fourth quarter of the cycle (unloading of the sample) is again characterized by the retardation of dislocations due to the increase in the number of obstacles. In the subsequent cycles, analogous phenomena take place; however, the number of dislocations will increase with the number of accumulated cycles.

Furthermore, by utilizing the concepts of the dislocation theory concerning the generation of vacanies in the process of motion and dislocation intersection (ref. 35), Oding assumed that the number of generated vacancies is proportional to the number of acting dislocations. The effectiveness of sample failure will depend on the effectiveness of combination of vacancies in a colony and the precipitation of vacancies on the surface of micropores. Vacancies originate during the motion of dislocations in planes of maximum tangential stresses (S-planes). If the normal stresses are absent in this plane then the formation of pores may take place only due to the combination of vacancies. The disintegration of a crystal lattice in volumes belonging to the S-planes is considered by the author as a result of the increase in the porosity of the metal due to the coagulation of vacancies. The increase of porosity in the S-planes leads to a local decrease in the strength of the metal; at the instant of time when the stress due to external forces is greater than the strength in the considered local volume, local failure occurs. When maximum normal stresses are present (N-plane), a great effectiveness, according to Oding, is achieved by the process of vacancy precipitation on the surface of micropores, /14 which leads to its regeneration into a crack. Depending on the values of both stresses, conditions are predetermined for the predominant development of coagulation processes or of processes associated with the precipitation of vacancies and also, as a result of this, the occurrence of failure along the S-plane or along the N-plane. These concepts have made it possible for the author to explain certain peculiarities of metals behavior under the action of assymmetric torsional cycles and also to find the relationship between stress, the number of cycles before failure, and the fatigue limit in the form:

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where N_i is the number of cycles before failure;

σ_i is the acting stress;

σ_N is the fatigue limit;

k_* and m are constant coefficients.

An analogous empirical equation was proposed by Weibull (ref. 37) on the basis of processing the experimental data obtained by Ravilly (ref. 38). The advantage of this equation compared with other equations is that it associates three important fatigue parameters (stress, number of cycles before failure, and fatigue limit) with a single relationship introduced on the basis of physical concepts regarding the fatigue process.

The thermal theory of fatigue proposed recently by Freudenthal and Weiner (ref. 39) is also rather interesting. The authors associate the formation of fatigue cracks with the action of local "temperature flashes" which occur in the slip planes under the action of cyclic stresses.

The presence of "temperature flashes" in the slip planes is confirmed both by the experimental data of Ludwik (ref. 40), which show that if the sample is cooled during the fatigue process its cyclic strength increases, as well as by the experiments of Hempel (ref. 41), which point to the occurrence of high-temperature processes during fatigue conditions (dispersion hardening, etc.).

In the opinion of Freudenthal and Weiner, the local heating during the action of a cyclic load is the result of multiple slippage along closely situated slip planes. The heat which is generated as a result of this is accumulated inside the metal regions which are enveloped with slip planes. When static loads act, this does not take place since the heat originating during the shear process is rapidly transmitted to other slip planes which are farther removed.

The repeated and variable stresses thus produce a large number of short term "temperature flashes", with a statistical distribution of intensity and locality inside the bands. The generated temperature in the local volumes will depend on the amplitude of the stress, on the frequency of cycles, heat conductivity and specific heat. The accumulating of heat inside the slip band during the presence of a large number of "temperature flashes" will lead to a gradual increase in the temperature and under favorable conditions it is possible to have a return, recrystallization and other processes which lead to the weakening of the metal. In addition to this the "temperature flashes" may lead to the generation of temperature gradients and consequently to the generation of thermal stress gradients. The magnitude of these stresses may reach values which are equal to the ultimate strength. Therefore, thermal stresses,

particularly when they are added to external stresses, are capable of producing microcracks which according to Freudenthal will probably originate /15
near the points where the slip bands meets the grain boundary.

Considering the slip process as a result of the motion of dislocations under the action of applied stress, Freudenthal and Weiner pointed out that the work done by external forces W is transformed into the potential energy of elastic distortion, W_e, the kinetic energy associated with the motion of dislocations W_k and dispersion energy W_d. Assuming that $W_e \ll W_d$ and noting W_e and W_k are small, it was assumed in the first approximation that

$$W_d \approx W = \tau nba,$$

where τ is the component of the applied stress in the slip plane;

a is the area described by the dislocation during its motion;

b is the Burgess vector;

n is the number of elementary shear displacements (which constitute a total shear by displacement of nb).

In this case the moving dislocation is looked upon as a moving source of heat.

Calculations have shown that in a technically pure annealed aluminum a local heating at the center of the slip band may reach a value of 150°, while in aluminum alloys it may reach a value of 555°.

Further, by analyzing the overall fatigue damage during one cycle caused by the action of "temperature flashes", the authors proposed an approximate relationship between the number of cycles before failure and various other parameters, in the form

$$N \cong [\text{const} \cdot f(k\rho c)\,\sigma_t]/[\tau(\tau - \tau_0)\,p_s E\alpha],$$

where k is the heat conductivity;

ρ is the density;

c is the specific heat;

α is the coefficient of linear heat expansion;

τ_0 is the critical shear stress;

τ is applied tangential stress;

p_s is the relative number of grains in which $\tau_0 < \tau$;

σ_t is the cohesive strength;

N is the number of cycles before failure.

The thermal fatigue theory is also quite interesting because it considers a complex of the physical properties of the metal which produce an undisputed important effect on the development of the fatigue process. However, from the point of view of this theory we should observe a substantial decrease in the fatigue limit as the frequency of cycles increases. However, experiment has shown the opposite to be true. There is a decrease in the fatigue limit as the frequency of cycles is increased (ref. 42). Furthermore, as shown by Cottrell (ref. 32) the value of the local temperature rise is insignificant and cannot lower the strength of the metal to a substantial degree in the considered microscopic volume.

Mott (ref. 43) has used the available experimental data to distinguish between three periods of fatigue:

(1) the period of strain hardening during which we have fine slippage (ref. 44) and the formation of bands;

(2) a period during which no further strain hardening is observed but the slip bands are expanded and become " unstable". Cracks develop in them (under certain definite conditions the extrusion of metal is possible);

(3) the period during which cracks develop which have formed in the second stage. The author has considered the reasons for the expansion of the slip bands (during the second stage of fatigue) and the formation of cracks in them. In this connection he used a model of slip band formation proposed /16
in the works of Friedel (refs. 45, 46) and others. According to Friedel a large number of short slip lines, as shown in figure 5, a, is formed in polycrystalline face-centered cubic metals and in monocrystals after the stage of fine slippage on the surface of the metal under the action of the Frank-Reed sources. In individual regions between the slip lines we may have transverse slippage (fig. 5e) which makes it possible for the dislocation sources S, S' and S'' to generate an ever-increasing number of dislocations and leads to the formation of slip bands.

During the continuous back-and-forth motion of dislocations along the line AB (fig. 6) which takes palce during cyclic loads and during their passage past dislocations situated along shorter lines CD, the transverse slippage will be responsible for the annihilation of the dislocations. This in turn leads to a softening of the metal because the source of dislocations S (fig. 6) and other closely situated sources will destroy a large number of dislocations.

The formation of fatigue cracks is explained by Mott in the same manner as by Fujita (ref. 31) as being the result of the annihilation of linear

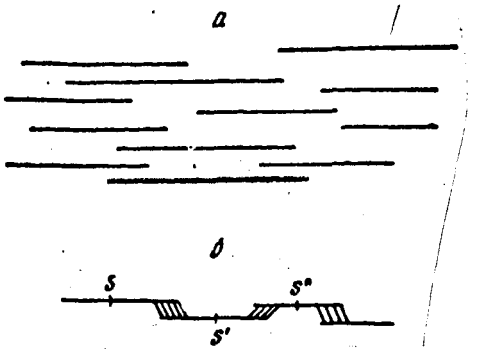

Figure 5. Schematic showing the formation of slip lines (ref. 43). a, formation of short slip lines in the first stages of strain hardening in face-centered monocrystals; b, formation of long lines by means of transverse slippage.

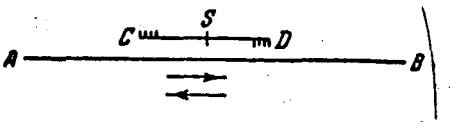

Figure 6. Schematic showing local weakening of metal under the action of cyclic stresses (ref. 43).

dislocations in parallel, closely situated slip planes. He points out that the temperature active process such as, for example, the chemical process (the action of oxygen) or the diffusion of vacancies may play a substantial role at high temperatures but are not significant for the fatigue of metals at normal and low temperatures. This conclusion is made by the author on the basis of data (ref. 47) obtained experimentally on fatigue at low temperatures.

At the same time Wood (ref. 48), in a report at the international London conference on the fatigue of metals, analyzed the theories of fatigue and noted that the formulation of the fatigue process from the point of view of the vacancy mechanism is by far the most plausible. In a subsequent work (ref. 49) he has shown that the mechanism of failure due to fatigue is different for large and small stresses. Large strain amplitudes produce strong strain hardening and the failure mechanism is associated with the formation of large accumulations of dislocations, i.e., it is analogous to static loading. For small stresses, on the other hand, the metal does not undergo strain hardening and breakdown is associated with the accumulation of vacancies in the apex regions of the slips formed during nonhomogeneous geometric slip.

The examination of the existing theories of fatigue makes it possible to conclude that at the present time there is still no single point of view regarding the process of fatigue failure. However, we should point out that the application of the theory of dislocations to the study of the fatigue failure mechanism has aided substantially in clarifying the nature of fatigue. In spite of the fact that the authors of the dislocation fatigue theory /17 approach the solution of the problem from various positions, they are united by a single idea which is reduced to the following. The factor which determines

the fatigue failure is the back-and-forth motion of dislocations and their interactions with each other and with other defects in the crystalline lattice. The effects which are associated with this (annihilation of dislocations, the origin of vacancies, local increase in stresses and temperature, etc.) are responsible for the origin of cracks.

The analysis of dislocation fatigue theories shows that under conditions when we have cyclic loads it is possible to have several mechanisms for the origin of fatigue cracks:

(1) as a result of the annihilation of dislocations (ref. 31),

(2) during the coagulation of vacancies (refs. 33, 34, 35, 48),

(3) in the process of the precipitation of vacancies at the apex of favorably oriented micropores (refs. 33, 34, 35),

(4) as a result of nonhomogeneous slip which leads to the extrusion and intrusion of metal (ref. 49).

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